References

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[3]Farassat, F., "Theory of Noise Generation from Moving Bodies with an Application to Helicopter Rotors," NASA TR R-451, Dec. 1975.

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Reply by Author to F. Farassat

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1) Dr. Farassat is totally mistaken to think that the right-hand sides of Eqs. (1) and (2) of Ref. 1 "must be zero." The right-hand sides of the mass and momentum conservation equations are zero only when there are no mass discontinuities and no prescribed body forces in the fluid medium. Attention of the discusser is drawn to Eq. (4) of Ref. 2, to Eq. (1.2) of Ref. 3, to the remark following Eq. (2.2.2) of Ref. 4, and to Eq. (6.8) of Ref. 5. A derivation of the continuity equation, Eq. (1) of Ref. 1, is given in the Appendix for the benefit of some readers. In fluid flow problems, it is up to an individual researcher whether to build the mass and force discontinuities into the conservation equations, as is done in the present derivation, or to account for them through boundary conditions as done in the earlier, Ffowcs Williams and Hawkings[6] and Goldstein[3] derivations. It so happens that inclusion of the mass and force distributions into the conservation equations in the present case leads to a simple, direct, physical derivation of the FW-H equation.

2) Writing the concentrated mass terms in Eq. (1), Ref. 1, in case of a moving surface, by the integral

$$\int_{f<0} \delta(x-\xi)\,\mathrm{d}m$$

is allowed by the fact that a surface is a continuum, and the very definition of a definite integral as the limit of a sum. Since almost everyone with some background in mathematics knows about such a basic definition, it hardly needs a justification or a reference. There are no gaps in the argument presented; the logic of the derivation is complete and perfectly valid within the framework of applied mathematics.

The reaction on the part of Dr. Farassat, in face of his personal efforts to obtain a simple derivation of the FW-H equation,[7] is a natural one.

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Appendix

Consider a closed surface S drawn in the region occupied by a moving fluid and fixed in space. Let $\mathrm{d}v$ be an element of the volume contained within S, ρ the density, and q the velocity of this element at any time t. Then, by the law of conservation of mass, the rate of change of mass of the fluid within the control surface S at any time is equal to the mass of the fluid that enters S per unit time plus the mass of the fluid created per unit time inside S due to the motion of the prescribed masses. This gives

$$\frac{\partial}{\partial t}\int_V \rho \mathrm{d}v = -\int_S q\cdot n \mathrm{d}S + \frac{\partial}{\partial t}\int_V \sum_{\xi} Q(\xi)\delta(x-\xi)\mathrm{d}v$$

where $\mathrm{d}S$ is an element of the surface S, n unit outward normal drawn at any point of $\mathrm{d}S$, V the volume of the fluid contained in S, Q the point mass located at he point $x=\xi(t)$ inside S, and $\delta(x)$ the three-dimensional Dirac delta function. Local time derivatives, $\partial/\partial t$, are used because S is fixed in space. Taking the time derivatives inside the integrals and using the divergence theorem gives

$$\int_V \left[\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho q) - \frac{\partial}{\partial t}\left\{\sum_{\xi} Q(\xi)\delta(x-\xi)\right\}\right]\mathrm{d}v = 0$$

Since V is arbitrary, it yields Eq. (1) of Ref. 1.

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[4]Batchelor, G. K., *Fluid Dynamics*, Cambridge University Press, London, 1967.

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